Long Monthly European Temperature Series and the North Atlantic Oscillation

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Introduction

Time series research on long temperature series (univariate)

- Harvey and Mills (2003) (the monthly Central England temperature (CET) series, 1723-2000; considerations based on time-aggregated quarterly and annual data)
- Vogelsang and Franses (2005) (the CET series, 1659-2000, and a Dutch (De Bilt) series, 1706-1993)
- Proietti and Hillebrand (2017) (the CET series, 1772-2013)
- He et al. (2019) (the CET series, 1772-2016)
- Hillebrand and Proietti (2017) (16 monthly European and 2 North American series, from the 18th century to (mostly) 2010s)

Introduction

Time series research on long temperature series (multivariate)

- He et al. (2021) (3 monthly European and 2 Chinese series (Beijing and Shanghai), around 1830-2018)
- He et al. (in press) (20 monthly European series, around 1750-2015)

Introduction

Perhaps the best known irregularly periodic climate phenomenon: the El Niño Southern Oscillation (ENSO) that has large effects on the weather in North and South America, and Australia.

- The strength of the ENSO is measured by the Southern Oscillation Index (SOI). The SOI is computed from fluctuations in the surface air pressure difference between Tahiti (in the Pacific) and Darwin, Australia (by the Indian Ocean)

Introduction

European counterpart of ENSO: the North Atlantic Oscillation (NAO) that affects the weather in Europe.

- The NAO index is based on the surface sea-level air pressure difference between the Subtropical (Azores) High and the Subpolar (Iceland) Low.
- There are different definitions of the index, available for different time periods.

Introduction

European counterpart of ENSO: the North Atlantic Oscillation (NAO) that affects the weather in Europe.

- The NAO index is based on the surface sea-level air pressure difference between the Subtropical (Azores) High and the Subpolar (Iceland) Low.
- There are different definitions of the index, available for different time periods.
- In this work, we attempt at building a nonlinear time series model for characterising effects of the NAO on European temperatures over time (1823–2015).

The VSSMC-AR-X model

The model is an extended version of the Vector Seasonal Shifting Mean and Covariance Autoregressive (VSSMC-AR) model (He et al., in press), called VSSMC-AR-X model.
Notation:

- $\mathbf{y}_{Sk+s} = (y_{1,Sk+s}, ..., y_{N,Sk+s})'$ is the $N \times 1$ vector of endogenous variables
- x_{Sk+s} the exogenous variable (can be generalised but only one exogenous variable in the application)
- $s = 1, ..., S$ denotes the season (in the application the month)
- $k = 0, 1, .., K - 1$, is the period (the year) counter
- $t = Sk + s$ is rescaled between zero and one, so the tth observation is indexed as $u_{ks} = (Sk + s)/SK$.

The VSSMC-AR-X model

Notation (continued)

- $D^{(j)}_{Sk+s}$ is a seasonal dummy variable: $D^{(j)}_{Sk+s} = 1$ for $j = s$, zero otherwise.
- To maintain this notation for observations lagged by h seasons (months), i.e., $t = Sk + s - h$, we adopt the modulo based equivalent representation $t = S\widetilde{k} + s_h$.
- Thus, $\widetilde{k} = \lfloor (Sk + s - h - 1)/S \rfloor$ for $k = 0, 1, \ldots, K - 1$, and $s_h = s - h \pmod{S}$. The residue system modulo S in this definition is the set $\{1, \ldots, S\}$.

The VSSMC-AR-X model

The mean equation

The mean equation of the VSSMC-AR-X model is defined as follows:

$$\begin{aligned}
\mathbf{y}_{Sk+s} &= \sum_{j=1}^{S}\{\delta_j(u_{kj}) + \boldsymbol{\phi}_{j0}x_{Sk+j}\}D_{Sk+s}^{(j)} + \sum_{h=1}^{p}\{\boldsymbol{\Phi}_h\mathbf{y}_{S\widetilde{k}+s_h} \\
&\quad + \sum_{j=1}^{S}\boldsymbol{\phi}_{jh}x_{S\widetilde{k}+j_h}D_{Sk+s}^{(j)}\} + \boldsymbol{\varepsilon}_{Sk+s} \\
&= \delta_s(u_{ks}) + \boldsymbol{\phi}_{s0}x_{Sk+s} + \sum_{h=1}^{p}\{\boldsymbol{\Phi}_h\mathbf{y}_{S\widetilde{k}+s_h} + \boldsymbol{\phi}_{sh}x_{S\widetilde{k}+s_h}\} \\
&\quad + \boldsymbol{\varepsilon}_{Sk+s}. \qquad (1)
\end{aligned}$$

The VSSMC-AR-X model

The mean equation

In (1),

- $\mathbf{\Phi}_h$ is an $N \times N$ parameter matrix, and
- $\boldsymbol{\phi}_{jh}$, $h = 0, 1, ..., p$; $j = 1, ..., S$, are $N \times 1$ parameter vectors.
- $\boldsymbol{\varepsilon}_{Sk+s}$ is the $N \times 1$ vector of independent errors with $\mathrm{E}\boldsymbol{\varepsilon}_{Sk+s} = \mathbf{0}$ and $\mathrm{E}\boldsymbol{\varepsilon}_{Sk+s} x_{Sk+s} = \mathbf{0}$.
- x_{Sk+s} is at least weakly exogenous to the parameters in (1).

The VSSMC-AR-X model

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- x_{Sk+s} is at least weakly exogenous to the parameters in (1).
- Roots of $|\mathbf{I}_N - \sum_{h=1}^{p} \mathbf{\Phi}_h z^h| = 0$ lie outside the unit circle.

The VSSMC-AR-X model

The mean equation

The deterministic time-varying intercept vector of the VSSMC-AR-X model for season s equals $\delta_s(u_{ks}) = (\delta_{s1}(u_{ks}), ..., \delta_{sN}(u_{ks}))'$, where the sth time-varying coefficient $\delta_{sl}(u_{ks})$ of equation l is defined as

$$\delta_{sl}(u_{ks}) = \delta_{sl0} + \sum_{i=1}^{q_{sl}} \delta_{sli} g_{sli}(u_{ks}; \gamma_{sli}, c_{sli}). \quad (2)$$

The VSSMC-AR-X model

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$$\delta_{sl}(u_{ks}) = \delta_{sl0} + \sum_{i=1}^{q_{sl}} \delta_{sli} g_{sli}(u_{ks}; \gamma_{sli}, c_{sli}). \tag{2}$$

In (2), the transition function is either logistic,

$$g_{sli}(u_{ks}; \gamma_{sli}, c_{sli}) = (1 + \exp\{-\gamma_{sli}(u_{ks} - c_{sli})\})^{-1} \tag{3}$$

or exponential,

$$g_{sli}(u_{ks}; \gamma_{sli}, c_{sli}) = 1 - \exp\{-\gamma_{sli}(u_{ks} - c_{sli})^2\} \tag{4}$$

In (3) and (4), $\gamma_{sli} > 0$, for $i = 1, ..., q_{sl}$ and $s = 1, ..., S$.

The VSSMC-AR-X model

The covariance equation

The error term $\boldsymbol{\varepsilon}_{Sk+s}$ of the VSSMC-AR-X model is decomposed as $\boldsymbol{\varepsilon}_{Sk+s} = \boldsymbol{\Sigma}_{Sk+s}^{1/2}\boldsymbol{\zeta}_{Sk+s}$, where $\boldsymbol{\zeta}_{Sk+s} \sim \mathrm{iid}(\mathbf{0}, \mathbf{I}_N)$, and

$$\boldsymbol{\Sigma}_{Sk+s} = \mathrm{E}\boldsymbol{\varepsilon}_{Sk+s}\boldsymbol{\varepsilon}'_{Sk+s} = \mathbf{D}_{Sk+s}\mathbf{P}_{Sk+s}\mathbf{D}_{Sk+s} \tag{5}$$

see Bollerslev (1990), where

- $\mathbf{D}_{Sk+s}$ is a diagonal matrix of standard deviations and
- $\mathbf{P}_{Sk+s}$ is a positive definite correlation matrix.

$\mathbf{D}_{Sk+s}$ and $\mathbf{P}_{Sk+s}$ may vary both with s and k.

The VSSMC-AR-X model

The covariance equation

Specifically, $\mathbf{D}_{Sk+s} = \mathrm{diag}(\sigma_{s1}(u_{ks}), \ldots, \sigma_{sN}(u_{ks}))$, where

$$\sigma_{sl}^2(u_{ks}) = \sigma_{sl0}^2 + \sum_{i=1}^{r_{sl}} \omega_{sli} g_{sli}(u_{ks}; \gamma_{sli}^{(v)}, c_{sli}^{(v)}) \qquad (6)$$

for $l = 1, \ldots, N$. In (6),

$$g_{sli}(u_{ks}; \gamma_{sli}^{(v)}, c_{sli}^{(v)}) = (1 + \exp\{-\gamma_{sli}^{(v)}(u_{ks} - c_{sli}^{(v)})\})^{-1} \qquad (7)$$

or

$$g_{sli}(u_{ks}; \gamma_{sli}^{(v)}, c_{sli}^{(v)}) = 1 - \exp\{-\gamma_{sli}^{(v)}(u_{ks} - c_{sli}^{(v)})^2\}. \qquad (8)$$

In (7) and (8), $\gamma_{sli}^{(v)} > 0$, $i = 1, \ldots, r_{sl}$; $s = 1, \ldots, S$.

The VSSMC-AR-X model

The covariance equation

Positivity condition for (6):

$$\sigma^2_{sl0} + \sum_{i=1}^{r_{sl}} \omega_{sli} g_{sli}(r; \gamma^{(v)}_{sli}, c^{(v)}_{sli}) > 0$$

for $\forall r \in [0, 1], s$ and k.

The VSSMC-AR-X model

The covariance equation

The time-varying error correlation matrix $\mathbf{P}_{Sk+s}$ in (5) for season s has the following form, see, for example, He et al. (2021),

$$\mathbf{P}_{Sk+s} = \{1 - g_s^{(\rho)}(u_{ks})\}\mathbf{P}_{(s1)} + g_s^{(\rho)}(u_{ks})\mathbf{P}_{(s2)}\} \qquad (9)$$

where $\mathbf{P}_{(s1)}$ and $\mathbf{P}_{(s2)}$ are $N \times N$ positive definite correlation matrices for season s.

The VSSMC-AR-X model

The covariance equation

In (9),

$$g_s^{(\rho)}(u_{ks}; \gamma_s^{(\rho)}, c_s^{(\rho)}) = (1 + \exp\{-\gamma_s^{(\rho)}(u_{ks} - c_s^{(\rho)})\})^{-1} \qquad (10)$$

or

$$g_s^{(\rho)}(u_{ks}; \gamma_s^{(\rho)}, c_s^{(\rho)}) = 1 - \exp\{-\gamma_s^{(\rho)}(u_{ks} - c_s^{(\rho)})^2\}. \qquad (11)$$

- Since both (10) and (11) are bounded between zero and one, as a convex combination of two positive definite correlation matrices $\mathbf{P}_{Sk+s}$ is positive definite for all $Sk + s$.

The VSSMC-AR-X model

The covariance equation

- NOTE: In the application, $\mathbf{P}_{Sk+s} = \mathbf{P}_s$.
- (Another simplification: $\boldsymbol{\Phi}_h$, $h = 1, ..., p$, in the mean equation are diagonal matrices.)

The VSSMC-AR-X model

Testing linearity of the mean equation

Before fitting the VSSMC-AR-X model to the data, it is necessary to test linearity.

- From the point of view of the application, one has to know whether or not the temperature series are nonstationary.
- From the statistical point of view, testing is necessary because the lth equation is not identified if the linearity hypothesis $\delta_{sl}(u_{ks}) = \delta_{sl0}$ in (2) holds for any $s = 1, ..., S$.

  Example:

$$\delta_{sl}(u_{ks}) = \delta_{sl0} + \delta_{sl1}(1 + \exp\{-\gamma_{sl1}(u_{ks} - c_{sl1})\})^{-1}.$$

The VSSMC-AR-X model

Testing linearity of the mean equation

- The test is based on approximating the alternative with a Taylor expansion around the null hypothesis $\gamma_{sli} = 0$, see Luukkonen et al. (1988).
- That is, testing zero transitions against one let H_0: $\gamma_{sl1} = 0$ and use the approximation (in this example a third-order one)

  $$\delta_{sl}(u_{ks}) \approx \delta^*_{sl0} + \delta^*_{sl1} u_{ks} + \delta^*_{sl2} u^2_{ks} + \delta^*_{sl3} u^3_{ks}.$$

  This results in a new null hypothesis H_0': $\delta^*_{sl1} = \delta^*_{sl2} = \delta^*_{sl3} = 0$. Can be tested by a standard χ^2-test.

The VSSMC-AR-X model

Testing linearity of the mean equation

Testing is carried out in stages as in He et al. (in press).

- First test the null hypothesis against one transition, i.e., $q_{sl} = 1$ in (2). Do this separately for $s = 1, ..., S$.
- If the null hypothesis is rejected for at least one s, estimate the equation with one transition and for the seasons with one transition, test against two.
- Proceed until the first non-rejection.

The VSSMC-AR-X model

ML estimation of parameters

- Estimate the mean equation first, then the variances and correlations.
- The variances and correlations are estimated jointly (no two-stage estimation).
  - Constancy of variances has to be tested before estimating the variance equations (identification problem).
- Asymptotic properties of ML estimators, see He et al. (in press).

The VSSMC-AR-X model

Joint estimation of variance and correlation parameters

Notation:

- $\boldsymbol{\theta}_s^{(v)} = (\boldsymbol{\theta}_{1s}^{(v)\prime}, \ldots, \boldsymbol{\theta}_{Ns}^{(v)\prime})'$, $s = 1, \ldots, S$, is the vector of variance parameters for season s, $\boldsymbol{\theta}_{ns}^{(v)}$, $n = 1, \ldots, N$, is the vector of parameters for season s in the nth equation
- $\boldsymbol{\theta}_s^{(c)}$ is the vector of parameters in the correlation matrix $\mathbf{P}_{Sk+s} = \mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)})$.

The VSSMC-AR-X model

Joint estimation of variance and correlation parameters

For season s, estimation proceeds as follows (Silvennoinen and Teräsvirta, in press).

1. Estimate the parameters in $\boldsymbol{\theta}_s^{(v)} = (\boldsymbol{\theta}_{1s}^{(v)\prime}, ..., \boldsymbol{\theta}_{Ns}^{(v)\prime})'$, $s = 1, ..., S$, equation by equation, assuming $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)}) = \mathbf{I}_N$. Denote the estimate $\widehat{\boldsymbol{\theta}}_s^{(v,1)} = (\widehat{\boldsymbol{\theta}}_{1s}^{(v,1)\prime}, ..., \widehat{\boldsymbol{\theta}}_{Ns}^{(v,1)\prime})'$. This means that the deterministic components $\sigma^2_{ns}(u_{ks})$, $n = 1, ..., N$, have been estimated once.

2. Estimate $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)})$, given $\boldsymbol{\theta}_s^{(v)} = \widehat{\boldsymbol{\theta}}_s^{(v,1)}$, $s = 1, ..., S$. This requires a separate iteration because $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)})$ is nonlinear in parameters, see (9), and (10) or (11). Denote the estimate $\mathbf{P}_{Sk+s}(\widehat{\boldsymbol{\theta}}_s^{(c,1)})$.

The VSSMC-AR-X model

Joint estimation of variance and correlation parameters

3. Re-estimate $\boldsymbol{\theta}_s^{(v)}$ assuming $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)}) = \mathbf{P}_{Sk+s}(\widehat{\boldsymbol{\theta}}_s^{(c,1)})$. This yields $\boldsymbol{\theta}_s^{(v)} = \widehat{\boldsymbol{\theta}}_s^{(v,2)}$. Then re-estimate $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)})$ given $\boldsymbol{\theta}_s^{(v)} = \widehat{\boldsymbol{\theta}}_s^{(v,2)}$. Iterate until convergence. Let the result after R_1 iterations be $\boldsymbol{\theta}_s^{(v)} = \widehat{\boldsymbol{\theta}}_s^{(v,R_1)}$ and $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)}) = \mathbf{P}_{Sk+s}(\widehat{\boldsymbol{\theta}}_s^{(c,R_1)})$. This completes the first iteration of the algorithm.

4. Re-estimate first the mean parameters given $\widehat{\boldsymbol{\theta}}_s^{(v,R_1)}$ and $\mathbf{P}_{Sk+s}(\widehat{\boldsymbol{\theta}}_s^{(c,R_1)})$, $s = 1, \ldots, S$, then jointly variance and correlation parameters using $\boldsymbol{\theta}_s^{(v)} = \widehat{\boldsymbol{\theta}}_s^{(v,R_1)}$ and $\mathbf{P}_{Sk+s}(\boldsymbol{\theta}_s^{(c)}) = \mathbf{P}_{Sk+s}(\widehat{\boldsymbol{\theta}}_s^{(c,R_1)})$ as starting values for the latter. This completes the second iteration of the algorithm.

5. Iterate until convergence (means, variances, correlations).

The data

28 weather stations in Europe, monthly time series for the years 1823–2015

- from Paris in the west to Kazan in the east,
- from Arkhangelsk in the north to Milan in the south,
- stations with data available from 1823 but with rather large numbers of missing variables were not included.

The data

28 weather stations in Europe, monthly time series for the years 1823–2015

- from Paris in the west to Kazan in the east,
- from Arkhangelsk in the north to Milan in the south,
- stations with data available from 1823 but with rather large numbers of missing variables were not included.
- (Tallinn not included: the series begins 1828 with gaps 1876–1880 and 1916–1920.)

The data

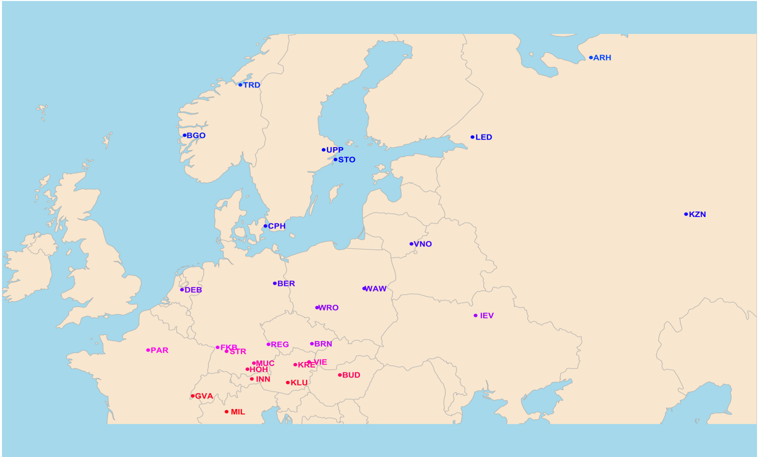


Figure: Map showing the locations of the 28 cities and towns from Arkhangelsk in the north to Milan in the south

Densities of temperature series

Trondheim, Uppsala, Stockholm, Copenhagen, Vilnius, Berlin, Warsaw and De Bilt

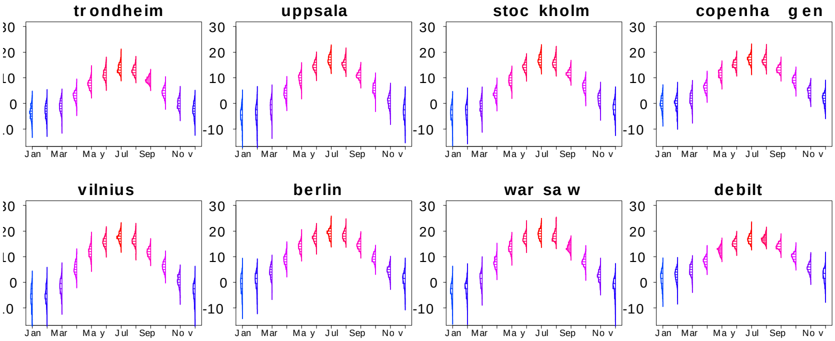


Figure: Densities (histograms) by month of eight temperature series, ordered by latitude, from Trondheim to De Bilt. Source: He et al. (in press)

The data

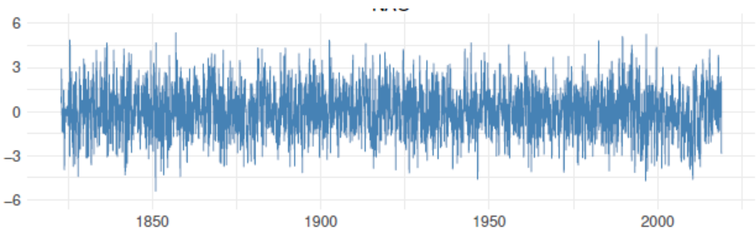


Figure: The monthly North Atlantic Oscillation index 1823–2015.
Source: Jones, Jónsson and Wheeler (1997), extended to 2015.

Results in the literature

$\mathbf{y}_t$ is nonstationary and dynamic
x_t is stationary (and dynamic)

- Hurrell and van Loon (1997), Hurrell (1996):
  Estimate β_1 from $y_t = \beta_0 + \beta_1 x_t + e_t$, where t is the aggregate for winter months
- Osborn (2011):
  Estimate β_1 from $y_t = \beta_0 + \beta_1 x_t + e_t$ by month (a separate equation for each month)
- Iles and Hegerl (2017):
  Estimate δ_1 from $x_t = \delta_0 + \delta_1 y_t + e'_t$ (they have a dense grid; they probably mean regressing y on x)

Results from the VSSMC-AR model (He et al., in press)

Shifting means by month, North

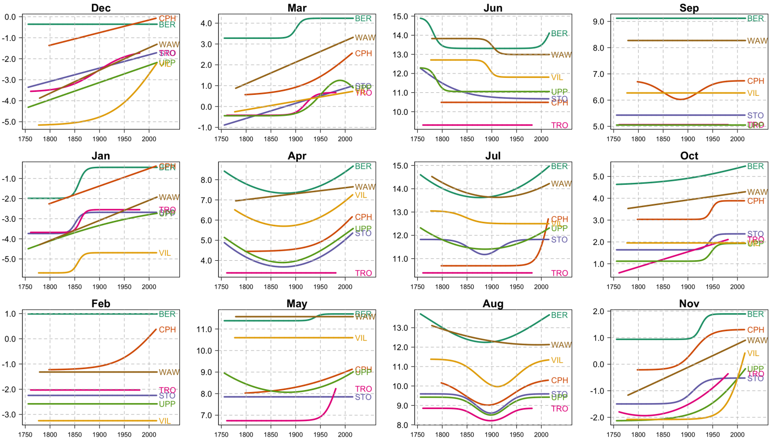


Figure: Estimated monthly temperature shifts for the locations in North. From north to south: Trondheim (TRO), Uppsala (UPP), Stockholm (STO), Copenhagen (CPH), Vilnius (VIL), Berlin (BER) and Warsaw (WAW).

Results from the VSSMC-AR model (He et al., in press)

Shifting means by month, West

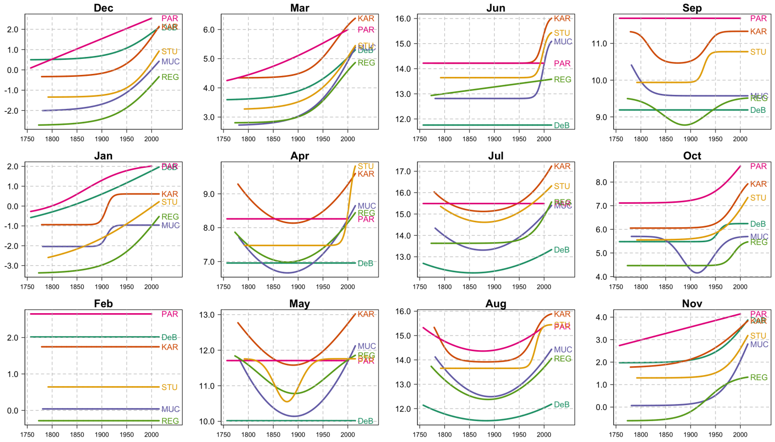


Figure: Estimated monthly temperature shifts for the locations in West. From west to east: Paris (PAR), De Bilt (DeB), Karlsruhe (KAR), Stuttgart (STU), Munich (MUN) and Regensburg (REG).

Results from the VSSMC-AR model (He et al., in press)

Shifting means by month, East-South

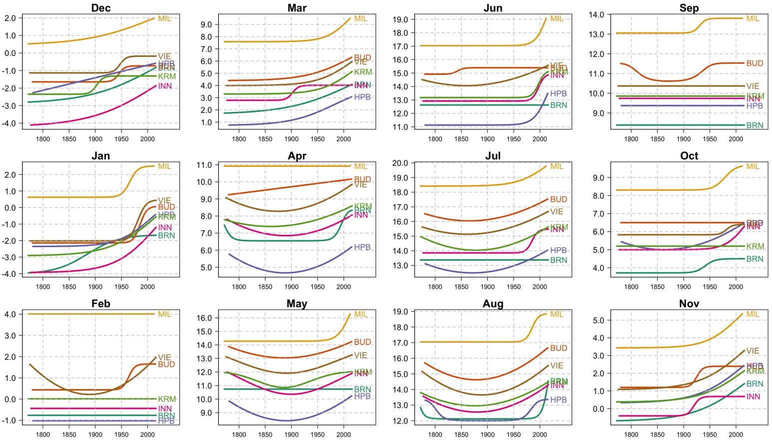
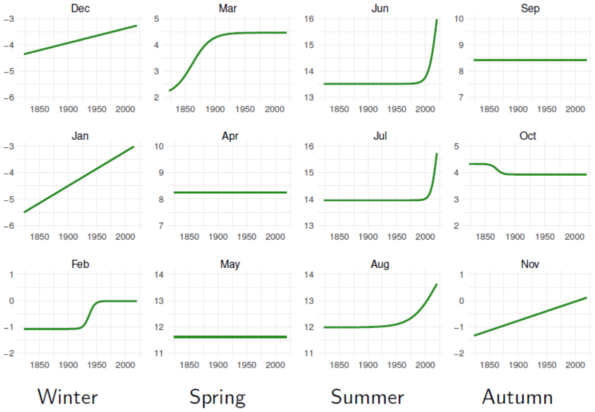


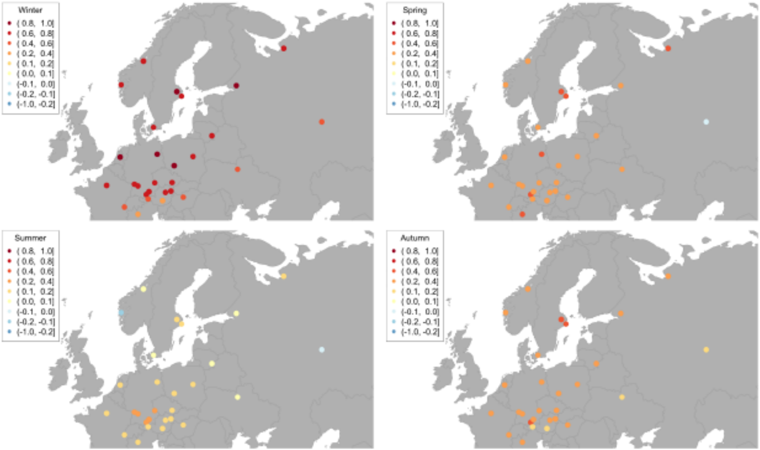
Figure: Estimated monthly temperature shifts for the locations in East/South. From west to east: Milan (MIL), Hohenpeissenberg (HPB), Innsbruck (INN), Kremsmünster (KRE), Vienna (VIE), Brno (BRN) and Budapest (BUD).

Results from the VSSMC-AR-X model

The shifting mean: Klagenfurt

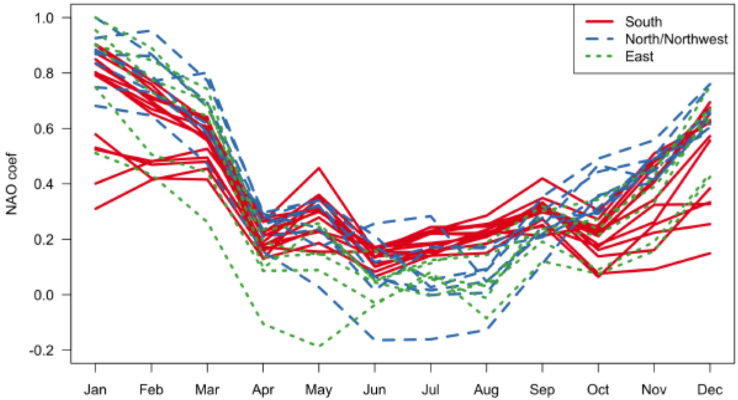


Results from the VSSMC-AR-X model



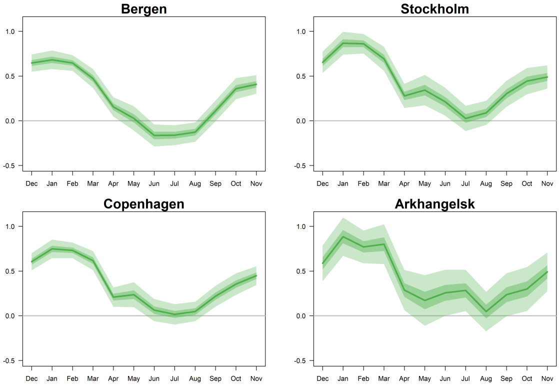
Seasonal averages of coefficent estimates of x_t . Top left: Winter (Dec-Feb), Top right: Spring (Mar-May), Bottom left: Summer (Jun-Aug), Bottom right. Autumn (Sep-Nov)

Results from the VSSMC-AR-X model



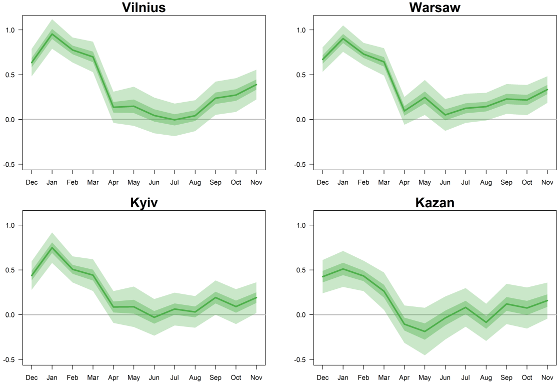
Estimates of the coefficient of x_t over the year in (1): Dashed lines: North/Northwest (Arkhangelsk, Bergen, Trondheim, Uppsala, Stockholm, Copenhagen, De Bilt, Berlin), Dotted lines: East (Kyiv, St Petersburg, Wroclaw, Warsaw, Vilnius, Kazan), Solid lines: South (the remaining 14 stations)

Results from the VSSMC-AR-X model



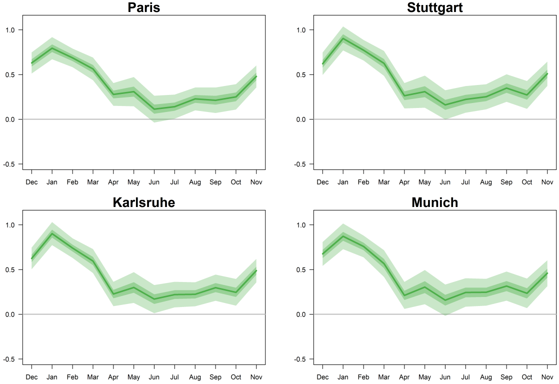
Estimates of the coefficient of x_t over the year in (1), solid line; 50% confidence level, dark green; 95% confidence level, light green. North/Northwest: Top panel: Bergen and Stockholm, Bottom panel: Copenhagen and Arkhangelsk

Results from the VSSMC-AR-X model



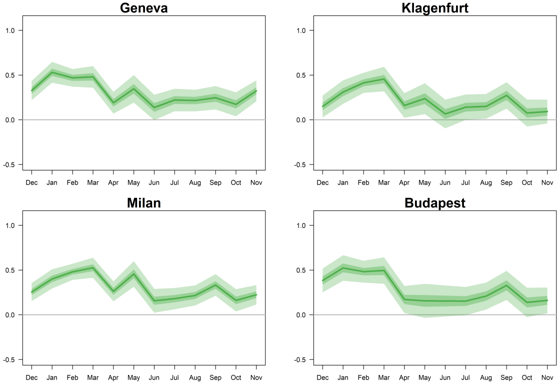
Estimates of the coefficient of x_t over the year in (1), solid line, 50% confidence level dark green, 95% confidence level, light green. North/Northeast: Top panel: Vilnius and Warsaw, Bottom panel: Kyiv and Kazan

Results from the VSSMC-AR-X model



Estimates of the coefficient of x_t over the year in (1), solid line; 50% confidence level, dark green; 95% confidence level, light green. West: Top panel: Paris and Stuttgart, Bottom panel: Karlsruhe and Munich

Results from the VSSMC-AR-X model



Estimates of the coefficient of x_t over the year in (1), solid line; 50% confidence level, dark green; 95% confidence level, light green. South/Southeast: Top panel: Geneva and Klagenfurt, Bottom panel: Milan and Budapest

Results from the VSSMC-AR-X model

Time-varying standard deviations

Standard deviation estimates: North (Arkhagelsk, Berlin, De Bilt, Trondheim, Copenhagen, Stockholm, Uppsala)

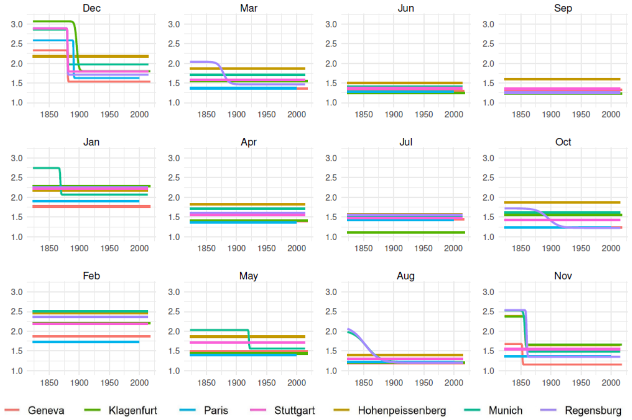
Results from the VSSMC-AR-X model

Time-varying standard deviations

Standard deviation estimates: Southeast (Brno-Turany, Innsbruck, Vienna, Budapest, Karlsruhe, Milan)

Results from the VSSMC-AR-X model

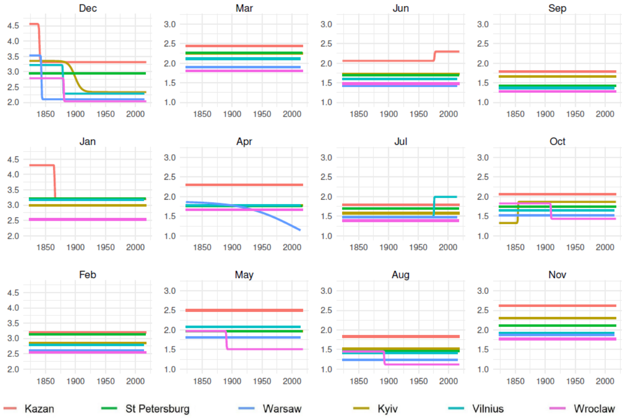
Time-varying standard deviations



Standard deviation estimates: Southwest (Geneve, Klagenfurt, Paris, Stuttgart, Hohenpeissenberg, Munich, Regensburg

Results from the VSSMC-AR-X model

Time-varying standard deviations



Standard deviation estimates: East (Kazan, St Petersburg, Warsaw, Kyiv, Vilnius, Wroclaw)

Error correlations: Spatial relationships

Generalise Haslett and Raftery (1989):

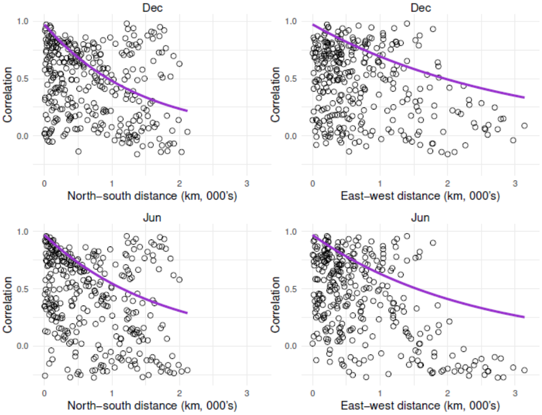
- d_{ij}^{NS} = latitudinal (north-south) distance between stations i and j
- d_{ij}^{EW} = longitudinal (east-west) distance between stations i and j
- d_{ij}^{H} = absolute difference in elevation between stations i and j
- $\widehat{\rho}_{sij}$ = residual correlation between stations i and j for month s

Minimise

$$Q_s^{(c)} = \min_{\boldsymbol{\theta}_s^{(c)}} (\widehat{\rho}_{sij} - \alpha_s \exp\{-(\beta_s^{NS} d_{ij}^{NS} + \beta_s^{EW} d_{ij}^{EW} + \beta_s^{H} d_{ij}^{H})\})^2$$

where α_s is the 'nugget effect' (Haslett and Raftery, 1989)

Error correlations: Spatial relationships



Estimated correlations and residual correlations against distance (in 1000K). (i) December: Top left: north-south, top right: east-west, (ii) June: Bottom left: north-south, bottom right: east-west

Error correlations: Spatial relationships

Correlations vs. absolute differences in elevation

- Statistically significant (but weak) relationship during the extended winter (Nov-Mar).
- No relationship for the remaining months.

Conclusions

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  - The winter effect is stronger in northern latitudes than in the south.
  - In the summer the effect is still significantly different from zero in the west, not so in the east.
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  - run counterfactuals (e.g., what happens to temperatures if instead of 'high' NAO there is 'low' NAO?)
- Distances between pairs of weather stations have a weak effect on error correlations.

Future work

- Climatologists know that the NAO also affects precipitation, at least in the winter.
- The VSSMC-AR-X model may be applied to investigating
  - seasonality, nonlinearity and nonstationarity of long monthly European precipitation series,
  - the effects of the NAO on precipitation using such series.
- This investigation has just begun.