Product versus Process Innovation: Evidence from Danish Producer-Level Data

Mons Chan
Queen's University

Amil Petrin
University of Minnesota and NBER

Frederic Warzynski
Stockholm School of Economics in Riga (SSE Riga) and CESifo

Bank of Estonia
November 5, 2026

Innovation can improve a product or change how it is made.

- A higher price may reflect greater willingness to pay, higher production costs, or a larger markup.
- Higher revenue productivity need not mean that a firm produces more physical output from its inputs.
- Do product and process R&D operate through different demand and cost channels?

We connect the purpose of firms' R&D spending to product-level demand, efficiency, costs, and prices.

Danish manufacturing data link R&D expenditure by objective to prices and physical quantities.

1. Recover product appeal from demand and physical efficiency from joint multiproduct production.
2. Estimate marginal costs and markups independently from demand and variable costs.
3. Relate these outcomes to lagged product, process, and general R&D.
4. Illustrate the product-appeal channel with a local, first-year welfare counterfactual.

Product R&D: higher appeal and higher cost.
Process R&D: greater physical efficiency and lower cost.

Trading Away Wide Brands for Cheap Brands, AER.

- Multiproduct firms jointly choose product and process innovation.
- Market expansion raises sales per product and the return to cost-reducing process investment.
- In the homogeneous-firm benchmark, import competition reduces product innovation as firms limit within-brand cannibalization.
- The product response can reverse when a wider brand improves visibility.

Initial productivity and export exposure shape these innovation responses.

Bilateral tariff cuts or a unilateral foreign tariff cut

Firm type	Product innovation	Process innovation
More productive exporters	Higher	More likely
Less productive exporters	Lower	More likely
Continuing non-exporters	Lower	Unchanged

- Exporters gain scale. Continuing non-exporters face import competition without market expansion.
- A home tariff cut reverses exporters' responses. Continuing non-exporters still reduce product innovation.

Aggregate R&D can conceal opposite product-innovation responses across firms.

- **Flach and Irlacher (2018):** multiproduct innovation choices depend on product differentiation and links in production costs.
- **Hallak and Sivadasan (2013):** product and process productivity are distinct determinants of quality choice and firm performance.
- **Cohen and Klepper (1996):** firm size shapes the allocation of R&D between product and process innovation.

We observe spending by objective and trace the corresponding demand and cost patterns.

- **Jaumandreu and Mairesse (2017); Jaumandreu and Lin (2018):** innovation, demand, costs, and price adjustment at the firm level.
- **Li, Li, and Zhang (2026):** the demand benefit and production cost of quality in Chinese steel.
- **Caselli, Chatterjee, and Li (2026):** joint recovery of product-level quality and productivity in multiproduct firms.

Our contribution

- Observe the allocation of R&D spending across objectives.
- Combine product-level demand with joint production, without assigning firm inputs by product sales shares.
- Compare demand- and cost-based marginal costs and markups.

Common analysis period: 1999–2016.

Source	Information used
VARS / PRODCOM	Firm-product sales and physical quantities; annual CN8 products
R&D survey	Total spending and allocation to product, process, and general R&D
FIRM accounts	Employment, wages, capital, investment, and materials
Customs, WITS, Comtrade	Imported-input prices, tariffs, and fixed trade weights
VARK input survey	Firm-level expenditures by input category

Outputs are observed by firm and product; inputs and R&D are observed at the firm level.

Measuring the allocation of R&D

- Product R&D improves or develops goods; process R&D improves production methods; general R&D is not assigned to either objective.
- Expenditure by objective equals total spending times the reported allocation share.
- The survey records total R&D annually and the allocation in alternating years.

Survey year	2001	2011	2016
Firms	3,536	4,424	4,522
Firms with positive R&D	566	983	1,095
Share with positive R&D	16.01%	22.22%	24.21%

Missing allocation years are not coded as zero spending. ▸ Coding and timing

Most R&D expenditure targets products

	(1) Product	(2) Process	(3) General
2001	74.79%	15.44%	9.77%
2003	76.09%	16.65%	7.26%
2005	80.76%	13.52%	5.72%
2007	74.32%	19.29%	6.39%
2009	69.26%	19.75%	10.99%
2011	75.07%	15.96%	8.96%

Expenditure-weighted shares, rather than shares of firms.

- Product R&D accounts for approximately three-quarters of spending.
- Process and general R&D remain economically meaningful margins.

R&D, prices, quantities, and product scope

	(1) Log Price	(2) Log Quantity	(3) Product count
Product R&D	0.388	-0.484	0.088
	(0.030)	(0.035)	(0.110)
Process R&D	-0.127	0.225	-0.084
	(0.036)	(0.042)	(0.143)
General R&D	0.014	0.054	0.294
	(0.034)	(0.040)	(0.140)
Observations	13,069	13,069	1,778

- Product R&D predicts higher prices and lower physical quantities.
- Process R&D predicts lower prices and higher physical quantities.
- Neither category has a precisely estimated product-count response.

$$\log s_{gjt} - \log s_{0dt} = \lambda_j + \lambda_g - \alpha_d p_{gjt} + \xi_{gjt}.$$

- Demand group d: CN6 product category and physical unit.
- Alternatives in each year: firm–CN8 products and an outside option.
- λ_j and λ_g: persistent firm and product components.
- ξ_{gjt}: time-varying product appeal, conditional on price.

Appeal can reflect quality and other demand attributes; price must be instrumented.

$s_{gjt} = Q_{gjt}/M_{dt}$ and $s_{0dt} = 1 - \sum_{gj \in d} s_{gjt}$, using the market-size normalization M_{dt}.

Use cost shifters while holding the firm's input bundle fixed.

1. **IV1:** market input prices, weighted by fixed input requirements.
2. **IV2:** input tariffs, using fixed input and country trade weights.
3. **IV3:** the input price series most correlated with the firm's product price, within its recorded input categories.

Identification requires input-price and tariff variation to be unrelated to unobserved product appeal, conditional on controls.

Fixed weights remove contemporaneous changes in input composition; they do not by themselves establish the exclusion restriction. ▸ Details

Input instruments imply more elastic demand

	OLS	Hausman	IV1	IV2	IV3	1+3	1+2+3
Panel A. Detailed Demand Estimation							
Example: Dining/Living Room Wooden Furniture (940360)							
Price sensitivity (α)	0.40	-5.8	4.0	3.9	4.1	4.7	4.2
	(0.02)	(7.11)	(1.12)	(0.51)	(0.61)	(0.62)	(0.55)
Obs.	1,328	1,328	551	551	351	351	351
F-Stat	-	0.78	74.4	56.2	45.3	29.5	20.7
Med. Elast.	-0.31	4.43	-3.07	-2.99	-3.10	-3.58	-3.20
Panel B. Overall (all 6-digit groups with ≥ 50 observations)							
Obs.	66,868	66,868	34,484	34,240	20,759	19,293	19,192
Med. Elast.	-0.35	-0.01	-2.65	-2.77	-2.71	-1.88	-1.53

The individual input instruments imply median elasticities near -3 in furniture and between -2.65 and -2.77 overall.

Standard errors in parentheses. Estimation samples differ across instruments. IV3 is used for the main demand-based decomposition.

$$q_{gjt} = \beta_0 + \beta_\ell \ell_{jt} + \beta_k k_{jt} + \beta_m m_{jt} + \gamma q_{-g,jt} + \omega_{gjt} + \eta_{gjt}.$$

- q_{gjt} is log physical output; ℓ, k, m are log firm inputs.
- $q_{-g,jt}$ aggregates the firm's other outputs using deflated revenue.
- The transformation function treats the firm as a joint producer.
- Investment and materials proxy for latent states; input prices enter the control function.

ω_{gjt} measures physical production efficiency, rather than quality-adjusted productivity.

Following Dhyne, Petrin, Smeets, and Warzynski (2022). ▸ Estimation ▸ Results

Recovering marginal cost from variable cost

$$\log VC_{jt} = f^{VC}\left(k_{jt}, \Delta k_{jt}, \overline{w}_{jt}, z^{I}_{jt}, q_{gjt}, q_{-g,jt}, \widehat{\omega}_{gjt}\right) + u_{gjt}.$$

- Flexible translog in quantities, input prices, capital, and efficiency.
- Capital adjustment allows quasi-fixed inputs to affect variable costs.
- Firm variable costs are not allocated across products using sales shares.

$$mc^{C}_{gjt} = \frac{VC_{jt}}{Q_{gjt}} \frac{\partial \log VC_{jt}}{\partial \log Q_{gjt}}, \qquad \mu^{C}_{gjt} = \frac{p_{gjt}}{mc^{C}_{gjt}}.$$

The derivative includes all own-output interactions. ▸ Results ▸ Derivative

Taking stock: four distinct outcomes

Object	Economic meaning
Product appeal ξ	Demand at a given price, net of persistent firm and product components
Physical efficiency ω	Physical output conditional on firm inputs and other products
Marginal cost mc	Cost of an additional physical unit
Markup $\mu = p/mc$	Price relative to marginal cost

A change in product quality can improve appeal while raising the cost of each physical unit.

Relating the outcomes to lagged R&D

For the state variables, product appeal and physical efficiency:

$$y_{gjt} = \kappa_y + \rho_y y_{gj,t-1} + \mathbf{R}_{j,t-1}\beta_y + \mathbf{X}_{gjt}\gamma_y + \tau_t + e^y_{gjt}.$$

- **R**: lagged log product, process, and general R&D spending.
- Controls: positive-spending indicators, product rank, product count, firm size, and year effects.
- Price, marginal-cost, and markup regressions use the same R&D variables and controls, without the lagged state term.

These are conditional relationships with prior R&D; the price instruments do not make R&D assignment exogenous.

Product R&D predicts higher product appeal

	(1) OLS	(2) Hausman	(3) IV1	(4) IV2	(5) IV3	(6) IV1+3	(7) IV1+2+3
Prod. R&D	-0.055	-0.011	0.463	0.474	0.552	0.363	0.229
	(0.031)	(0.076)	(0.188)	(0.191)	(0.211)	(0.155)	(0.140)
Proc. R&D	-0.009	-0.048	-0.456	0.078	0.049	0.156	0.145
	(0.038)	(0.093)	(0.238)	(0.244)	(0.269)	(0.197)	(0.179)
Gen. R&D	0.115	0.065	-0.254	-0.508	-0.446	-0.357	-0.205
	(0.038)	(0.094)	(0.240)	(0.246)	(0.271)	(0.198)	(0.180)
Lag appeal ($\xi_{gj,t-1}$)	0.919	0.807	0.976	0.980	0.876	0.886	0.853
	(0.011)	(0.009)	(0.004)	(0.005)	(0.013)	(0.015)	(0.019)
Observations	2,956	2,954	2,323	2,361	1,498	1,503	1,499

Product R&D is positive across all input-IV specifications. Process R&D has no stable positive association with appeal.

Firm-clustered standard errors in parentheses. The dependent variable is appeal in utility units: coefficients are not percentage changes in quality.

$$\varepsilon^D_{gjt} = -\alpha_d p_{gjt}(1 - s_{gjt}), \qquad \mu^D_{gjt} = \frac{\varepsilon^D_{gjt}}{1 + \varepsilon^D_{gjt}}, \qquad mc^D_{gjt} = \frac{p_{gjt}}{\mu^D_{gjt}}.$$

- The pricing condition applies to single-product Bertrand pricing, or negligible cross-price effects among a firm's own products.
- Retain economically admissible elasticities on the elastic part of demand.
- Decompose the price relationship using

$$\log p_{gjt} = \log mc^D_{gjt} + \log \mu^D_{gjt}.$$

Does R&D predict a higher price because costs rise, or because the firm charges a higher markup?

Demand-based price decomposition

	(1) $\log p_{gjt}$	(2) $\log mc^D_{gjt}$	(3) $\log \mu^D_{gjt}$	(4) $\log(-\varepsilon^D_{gjt})$
Prod. R&D	0.550	0.477	0.073	-0.207
	(0.098)	(0.094)	(0.029)	(0.074)
Proc. R&D	-0.343	-0.317	-0.026	0.065
	(0.124)	(0.119)	(0.038)	(0.100)
Gen. R&D	0.066	-0.056	0.122	-0.353
	(0.125)	(0.120)	(0.038)	(0.101)
Observations	1,241	1,241	1,241	1,234

- Product R&D: higher marginal cost accounts for about 87% of the price coefficient; the markup contribution is smaller.
- Process R&D: the price decline is largely accounted for by lower marginal cost.

IV3 demand objects; lagged log R&D and controls. Firm-clustered standard errors in parentheses.

Cost-based price decomposition

	(1) $\log p_{gjt}$	(2) $\log mc^{C}_{gjt}$	(3) $\log \mu^{C}_{gjt}$	(4) ω_{gjt}
Prod. R&D	0.678	0.769	-0.089	-0.227
	(0.067)	(0.068)	(0.025)	(0.079)
Proc. R&D	-0.377	-0.263	-0.119	0.372
	(0.084)	(0.085)	(0.031)	(0.098)
Gen. R&D	0.081	0.072	0.010	-0.102
	(0.061)	(0.081)	(0.022)	(0.069)
Observations	3,589	3,589	3,589	3,292

- Product R&D: higher cost and lower measured physical efficiency.
- Process R&D: lower cost and greater physical efficiency.
- Markup responses differ from the demand decomposition.

Variable-cost estimates; lagged log R&D and controls. Firm-clustered standard errors in parentheses. Samples differ from the demand approach.

Product innovation

- Makes products more attractive to buyers.
- Can require more expensive inputs or production methods.
- A higher price and lower physical efficiency can accompany upgrading.

Process innovation

- Produces more physical output from the firm's inputs.
- Lowers marginal cost and product prices.
- Has little systematic positive relationship with product appeal.

Demand and cost estimates agree on this distinction, while the markup channel is less uniform.

Raise product R&D by 1% among positive investors in 2011.

1. Shift treated products' appeal by $\Delta\xi = \widehat{\beta}^{\xi}_{\text{prod}} \log(1.01)$.
2. Recompute logit shares and consumer surplus exactly.
3. Compute changes in investing firms' revenue and variable costs.
4. Deduct the additional R&D expenditure once per firm.

Hold prices and unit marginal costs fixed to isolate the first-year demand channel.

A local channel calculation, rather than an estimated response to an R&D subsidy. ▸ Formulas

Private and consumer returns differ

First-year change	Million DKK	Per extra DKK
Consumer surplus	11.6	1.5
Investing-firm operating profit	6.0	0.8
Additional product R&D cost	8.0	1.0
Investing-firm producer surplus	−2.0	−0.3
Reported total surplus	**9.6**	**1.2**

Consumers capture benefits that the investing firms do not fully appropriate.

Total surplus is consumer surplus plus investing-firm producer surplus, net of additional R&D. Values are rounded independently; calculate ratios from unrounded results. ▸ Industry levels

Returns vary across industries

	CN2 industry					
	62	73	84	85	94	Total
First-year effects (DKK per additional DKK of product R&D)						
Consumer surplus	3.6	2.8	1.8	0.6	0.8	1.5
Operating profit	1.4	0.3	1.1	0.4	0.3	0.8
Additional product R&D expenditure	1.0	1.0	1.0	1.0	1.0	1.0
Investing-firm producer surplus	0.4	-0.8	0.1	-0.6	-0.7	-0.3
Total surplus	**4.0**	**2.0**	**1.8**	**0.0**	**0.1**	**1.2**
Gross social benefit-cost ratio	5.0	3.0	2.8	1.0	1.1	2.2

The reported first-year net return is positive in four industries and approximately zero in electrical machinery.

CN2: 62 apparel (not knitted); 73 iron/steel articles; 84 machinery; 85 electrical machinery; 94 furniture. Values are rounded independently.

- The demand channel creates a gap between consumer gains and the investing firm's private return.
- The calculation is consistent with underinvestment along the product-appeal margin under the maintained assumptions.
- It does not model endogenous R&D responses to subsidies, equilibrium repricing, entry, or the financing of public support.
- It also omits a complete accounting of competitors' profit changes and longer-run spillovers.

The exercise measures one economically relevant channel; it does not identify an optimal subsidy rate.

1. **The purpose of R&D matters.**
   Product and process spending predict different demand and cost outcomes.

2. **Physical efficiency alone misses product upgrading.**
   Higher appeal can accompany higher marginal cost and lower physical output per input.

3. **Two approaches support the central cost distinction.**
   Demand- and cost-based markups provide complementary, but not identical, decompositions.

4. **Consumer gains can exceed private gains.**
   The local calculation highlights the product-appeal channel.

Fixed input requirements

Let H_{jt} be firm j's observed input set in year t. Define

$$H_j = \bigcup_t H_{jt}, \qquad \overline{Q}^I_{hgj} = \frac{\sum_t Q^I_{hjt}}{\sum_t Q_{gjt}}.$$

- The weight measures units of input h per unit of output g over the sample.
- The union fixes the input set; the ratio fixes input requirements.
- Market input-price changes then generate variation, rather than current changes in the firm's input mix.

Remaining identifying assumption: these changes are excluded from time-varying product appeal.

◂ Back

Market input prices and fixed requirements

$$p^I_{ht} = \sum_j w_{hjt} p^I_{hjt}, \qquad z^1_{gjt} = \sum_{h \in H_j} \overline{Q}^I_{hgj} p^I_{ht}.$$

w_{hjt} is the importing firm's quantity share for input h in year t.

Tariffs and fixed country weights

$$\tau_{ht} = \sum_c \lambda_{hc} \tau_{hct}, \qquad z^2_{gjt} = \sum_{h \in H_j} \overline{Q}^I_{hgj} \tau_{ht}.$$

- λ_{hc} uses 1999 global trade weights; tariffs are EU applied rates.
- The preferred tariff measure uses a positive-tariff indicator.

◂ Back

IV3: SELECTING AN INFORMATIVE INPUT PRICE

Within the CN6 input categories recorded for firm j, consider the available CN8 market input-price series:

$$h^*_{gj} = \arg\max_{h \in H^8_j} \mathrm{corr}_t(p^I_{ht}, p_{gjt}), \qquad z^3_{gjt} = \overline{Q}^I_{h^* gj} p^I_{h^* t}.$$

- Select separately for each firm-product pair.
- Retain the market-level source of input-price variation.
- Target the input series that best predicts the corresponding output price.

Predictive relevance does not replace the exclusion restriction.

◂ Back

Estimating multiproduct physical efficiency

- Estimate the transformation equation by broad CN2 industry using firm-product observations.
- Investment and materials provide two proxies for latent efficiency states, conditional on capital and input prices.
- Identification requires joint invertibility of these proxy decisions.
- Use Wooldridge-style moment conditions with lagged proxies and other output, and current and lagged capital.
- Recover $\widehat{\omega}_{gjt}$ for the R&D regressions and as a control in the variable cost function.

The other-output index is normalized to zero for single-product firms.

◂ Back

Multiproduct production estimates

	Production Function Estimates					
	Meat Products 16	Plastics 39	Fab. Metal 73	Machinery & Equip. 84	Electrical Machinery 85	Furniture 94
$\ln Q_{-g}$	-0.171	-0.095	-0.100	-0.112	-0.082	-0.135
	(0.012)	(0.006)	(0.005)	(0.004)	(0.008)	(0.004)
ℓ	0.320	-0.095	-0.042	0.180	0.216	0.371
	(0.065)	(0.074)	(0.052)	(0.049)	(0.097)	(0.048)
k	0.113	0.317	0.187	0.055	0.191	0.145
	(0.037)	(0.036)	(0.024)	(0.018)	(0.037)	(0.019)
m	0.540	0.762	0.723	0.731	0.419	0.651
	(0.047)	(0.059)	(0.038)	(0.042)	(0.075)	(0.037)
Obs.	1,412	2,539	3,942	7,411	2,032	3,957

Other-output coefficients are negative: holding firm inputs fixed, producing more of the other products reduces physical output of product g.

Standard errors in parentheses.

◂ Back

Selected variable-cost coefficients

	Cost Function Estimates	
	Variable Cost	(s.e.)
Quantity	0.28	(0.07)
Quantity^2	0.02	(0.001)
Capital	-1.30	(0.10)
Δ Capital	0.08	(0.01)
Mean Wage	2.56	(0.36)
Input Prices	0.13	(0.06)
Obs.	15,565	

The full translog includes other output, physical efficiency, squares, and interactions; these are selected coefficients.

Standard errors in parentheses. First-order coefficients alone are not the cost elasticities at every observation.

◂ Back

Write the variable-cost specification schematically as

$$\log VC = a_0 + \sum_r a_r x_r + \sum_r b_{rr} x_r^2 + \sum_{r<s} b_{rs} x_r x_s + a_\Delta \Delta k + u,$$

where x includes log own output q, other output, capital, input prices, and physical efficiency. Then

$$\frac{\partial \log VC}{\partial q} = a_q + 2 b_{qq} q + \sum_{r \neq q} b_{qr} x_r, \qquad mc^C = \frac{VC}{Q_g} \frac{\partial \log VC}{\partial q}.$$

Firm-level cost is differentiated with respect to one product's output, holding the other arguments fixed.

◂ Back

For each innovation objective a:

$$D_{jt}^{a} = \mathbf{1}\{RD_{jt}^{a} > 0\}, \qquad r_{jt}^{a} = \begin{cases} \log RD_{jt}^{a}, & RD_{jt}^{a} > 0, \\ 0, & RD_{jt}^{a} = 0. \end{cases}$$

- Include D^{a} separately, so the log-spending slope describes the intensive margin among positive spenders.
- Use one-period lagged spending in each objective.
- Do not turn unobserved allocation years into genuine zero R&D.
- Add lagged appeal or physical efficiency when studying those state variables.

◂ Back

Exact logit updating in the local exercise

For market m, let T_m be the treated products and $S_{T,m}$ their initial share. All receive the same appeal shift δ:

$$\delta = \widehat{\beta}^{\xi}_{\text{prod}} \log(1.01), \qquad D_m = 1 + (e^{\delta} - 1) S_{T,m}.$$

$$s'_i = \begin{cases} s_i e^{\delta} / D_m, & i \in T_m, \\ s_i / D_m, & i \notin T_m, \end{cases} \qquad s'_0 = s_0 / D_m.$$

$$\Delta CS_m = \frac{M_m}{\alpha_m} \log D_m, \qquad \Delta Q_i = M_m (s'_i - s_i).$$

$$\Delta PS_T = \sum_{i \in T} (p_i - mc_i^C) \Delta Q_i - \Delta RD, \qquad \Delta W = \sum_m \Delta CS_m + \Delta PS_T.$$

Fixed prices and unit marginal costs. R&D costs counted once per investing firm; the reported producer-surplus term covers investing firms.

◂ Back

Million DKK; 1% more product R&D in 2011.

	CN2 industry					
	62	73	84	85	94	Total
Consumer surplus (ΔCS)	1.8	1.1	6.5	1.2	1.0	11.6
Revenue (ΔR)	4.8	2.9	17.9	3.1	2.8	31.5
Variable production cost (ΔVC)	4.1	2.8	14.0	2.2	2.4	25.5
Operating profit ($\Delta \Pi^{op}$)	0.7	0.1	3.9	0.9	0.4	6.0
Additional product R&D expenditure (ΔRD)	0.5	0.4	3.7	2.1	1.3	8.0
Investing-firm producer surplus (ΔPS)	0.2	-0.3	0.2	-1.2	-0.9	-2.0
Total surplus (ΔW)	**2.0**	**0.8**	**6.7**	**0.0**	**0.1**	**9.6**

CN2: 62 apparel; 73 iron/steel articles; 84 machinery; 85 electrical machinery; 94 furniture. Prices and unit marginal costs held fixed; values rounded.

◂ Back

Selected references: innovation choices

- Cohen, W. M., and S. Klepper (1996). "Firm Size and the Nature of Innovation within Industries: The Case of Process and Product R&D." *Review of Economics and Statistics* 78(2): 232–243.

- Dhingra, S. (2013). "Trading Away Wide Brands for Cheap Brands." *American Economic Review* 103(6): 2554–2584. doi:10.1257/aer.103.6.2554.

- Flach, L., and M. Irlacher (2018). "Product versus Process: Innovation Strategies of Multiproduct Firms." *AEJ: Microeconomics* 10(1): 236–277.

- Hallak, J. C., and J. Sivadasan (2013). "Product and Process Productivity: Implications for Quality Choice and Conditional Exporter Premia." *Journal of International Economics* 91(1): 53–67.

Selected references: demand, costs, and quality

- Jaumandreu, J., and J. Mairesse (2017). "Disentangling the Effects of Process and Product Innovation on Cost and Demand." *Economics of Innovation and New Technology* 26(1–2): 150–167.

- Jaumandreu, J., and S. Lin (2018). "Prices under Innovation: Evidence from Manufacturing Firms." CEPR Discussion Paper 13146.

- Li, J., S. Li, and H. Zhang (2026). "Output Quality, Productivity, and Demand: Evidence from the Chinese Steel Industry." *RAND Journal of Economics* 57(3): 784–807. doi:10.1111/1756-2171.70063.

- Caselli, M., A. Chatterjee, and S. Li (2026). "Productivity and Quality of Multi-product Firms." International Finance Discussion Paper 1430, Federal Reserve Board.

Selected references: multiproduct estimation

- Dhyne, E., A. Petrin, V. Smeets, and F. Warzynski (2022). "Theory for Extending Single-Product Production Function Estimation to Multi-Product Settings." NBER Working Paper 30784. doi:10.3386/w30784.

- Wooldridge, J. M. (2009). "On Estimating Firm-Level Production Functions Using Proxy Variables to Control for Unobservables." *Economics Letters* 104(3): 112–114.

Further references and full specifications are in the accompanying paper.